

# **A complete basis set model chemistry for excited states**

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# Outline

## **I. Background**

- a. John Pople's Model Chemistries

## **II. Excited state Models**

- a. CCSD(T) vs FCI
- b. CASSCF and CAS-CISD vs FCI

## **III. CBS Extrapolations**

- a. A Systematic Sequence of Basis Sets
- b. The CASSCF CBS Limit
- c. The CASSCF-CISD CBS Limit

## **IV. Results**

- a. Geometry
- b. Vibrational Frequencies
- c. Excitation energies
- d. Dissociation Energies

## **V. Acknowledgment**

# John Pople's Model Chemistries

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## The CBS-Q//B3 Model

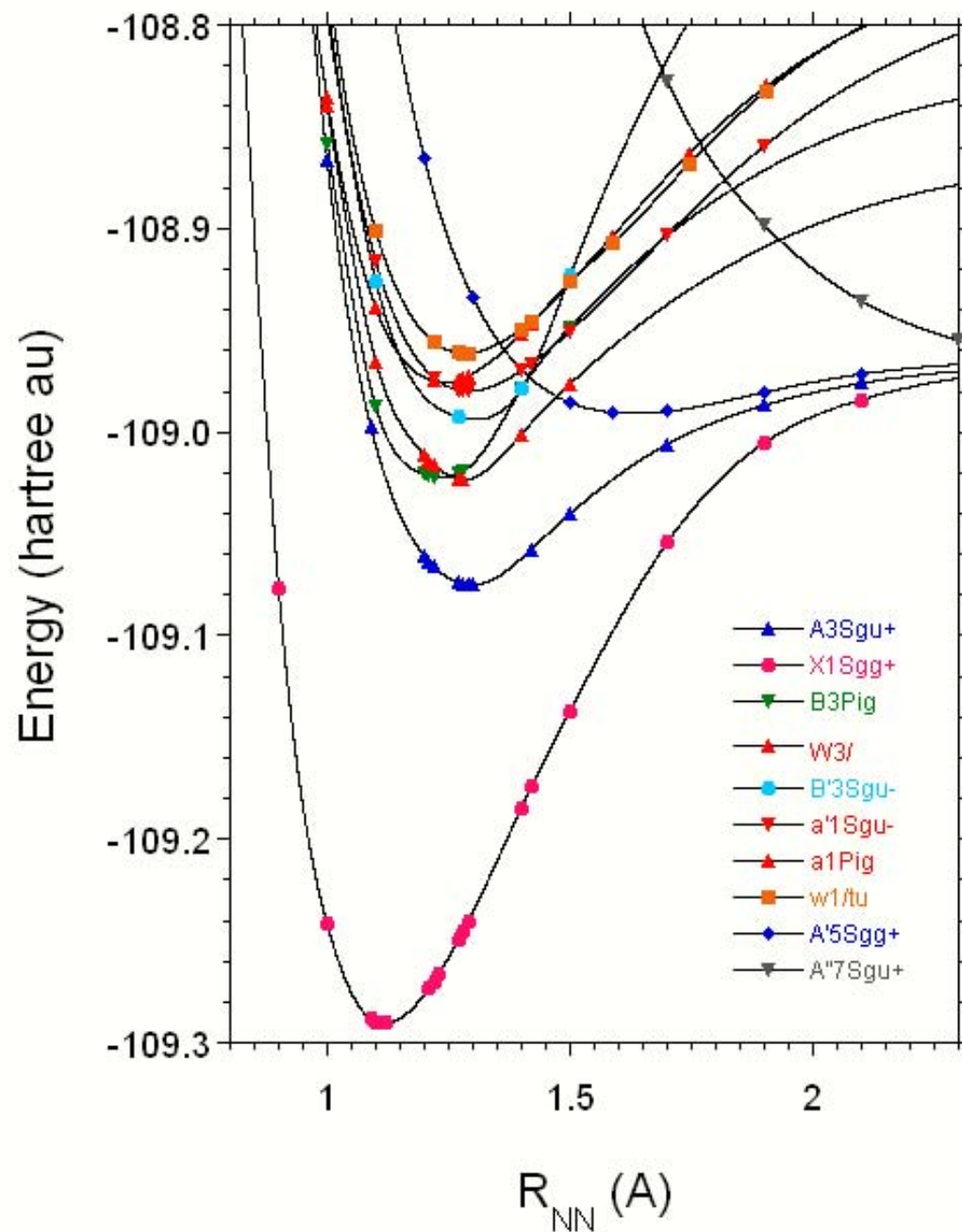
**Geometry:** B3LYP/6-311G(2d,d,p)  
**ZPE & Thermal:** B3LYP/6-311G(2d,d,p)  
**SCF:** UHF/6-311+G(3d2f,2df,2p)  
**CBS(extrap) - MP2:** UHF/6-311+G(3d2f,2df,2p)  
**MP3, MP4(SDQ):** MP4(SDQ)/6-31+G(d(f),d,p)  
**MP5 - CCSD(T):** CCSD(T)/6-31+G(d')

### Empirical Corrections:

$-5.79 \text{ mE}_h |S|^2_{ii} [\Sigma_i C^2_{\mu}]_{ii} \quad (2e^-)$   
 $-9.54 \text{ mE}_h \Delta(S^2) \quad (\text{Spin Contam.})$   
 $E(\text{core}) \sim 3.92 \text{ mE}_h Q_{\text{Na}} + 2.83 \text{ mE}_h Q^2_{\text{Na}}$   
Experimental Atomic Spin-Orbit Interaction

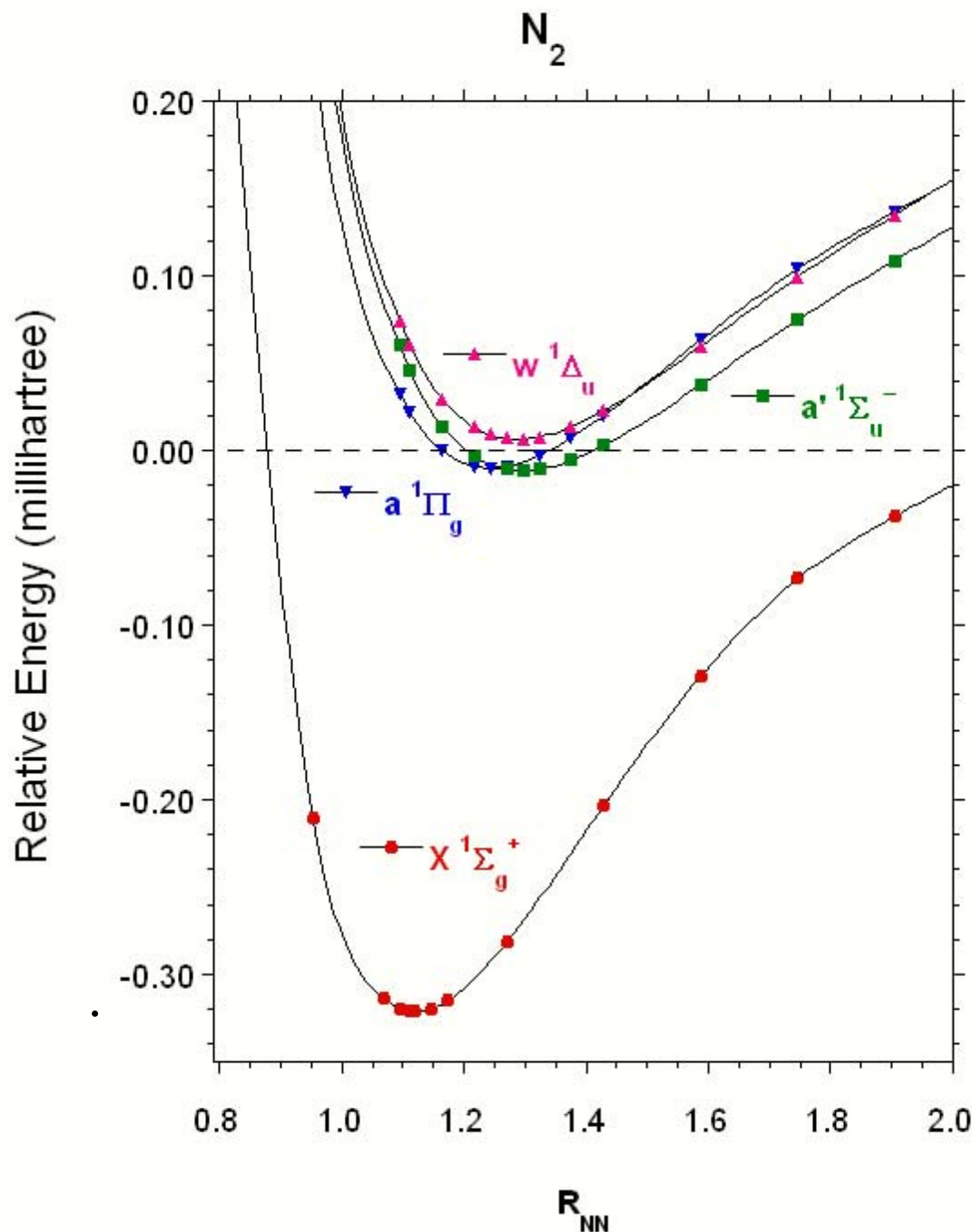
## Test Case

The  $X^1\Sigma_g^+$  ground state, along with the  $a^3\Sigma_u^+$ ,  $A'^5\Sigma_g^+$ , and  $A''^7\Sigma_u^+$  excited states of  $\text{N}_2$  dissociates to two  $^4S_{3/2}$  ground state N atoms. Other low-lying singlet and triplet states dissociate to N atoms in  $^2D$  and  $^2P$  excited states.



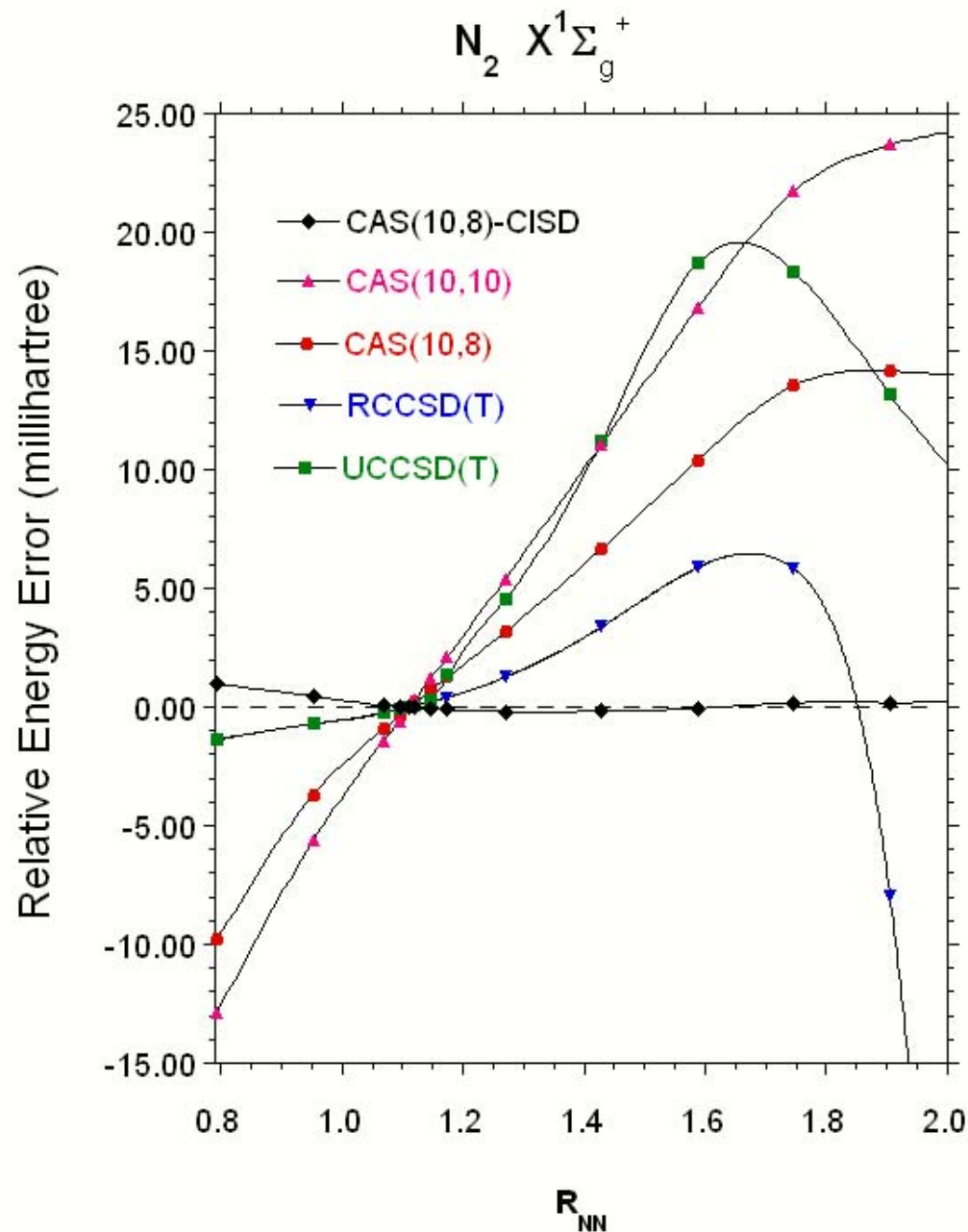
## Benchmarks

The **FCI/cc-pVDZ** energies of low-lying states of **N<sub>2</sub>** [H. Larsen, J. Olsen, P Jørgensen, and O. Christiansen, *J. Chem. Phys.* **113**, 6677 (2000)] provide benchmarks for calibration of model chemistry candidates.



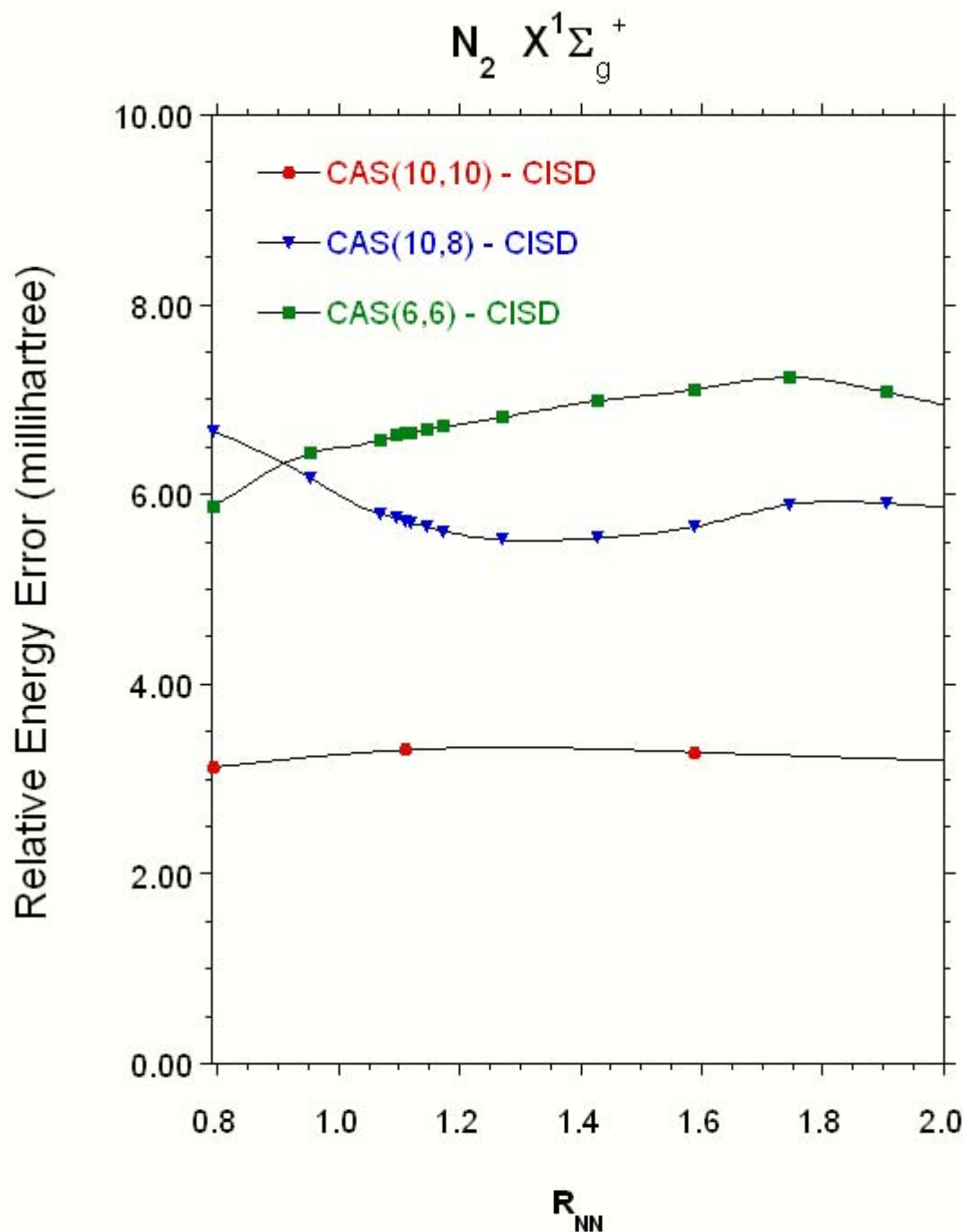
# Model

The **error** in the energy of the ground state of  $\text{N}_2$ , relative to the energy at the equilibrium bond length. Note that the Multi-reference **CAS-CISD** is far more accurate than **CCSD(T)**.





The error in the energy of the ground state of  $N_2$ , relative to the Full CI energy is insensitive to bond length, but a **constant size-consistency correction** must be made for comparisons with atoms.



**Table I.** The effect of **state averaging** on CASCF and CASSCF-CISD

State	R <sub>NN</sub> (Å)	Basis Set	N <sub>root</sub> =1 (E <sub>h</sub> )	N <sub>root</sub> =6 (E <sub>h</sub> )	ΔE (mE <sub>h</sub> )
CAS(6 <sub>e-</sub> ,6 <sub>Orb</sub> )					
X <sup>1</sup> Σ <sub>g</sub> <sup>+</sup>	1.10	2ZaP	-109.1045238	-109.0781418	26.3820
		3ZaP	-109.1251169	-109.0979344	27.1825
A <sup>3</sup> Σ <sub>u</sub> <sup>+</sup>	1.27	2ZaP	-108.8740932	-108.8606554	13.4378
		3ZaP	-108.8898435	-108.8761258	13.7177
CAS(6 <sub>e-</sub> ,6 <sub>Orb</sub> ) – CISD					
X <sup>1</sup> Σ <sub>g</sub> <sup>+</sup>	1.10	2ZaP	-109.2896344	-109.2885919	1.0425
A <sup>3</sup> Σ <sub>u</sub> <sup>+</sup>	1.27	2ZaP	-109.0736612	-109.0733565	0.3047

**Table II.** Calculated N<sub>2</sub> cc-pVDZ **excitaton energies** (mE<sub>h</sub>).

RNN(Å)	Excitation	CAS(6,6)	CAS(6,6)- CISD	FCI
<b>1.1113</b>	$X^1\Sigma_g^+ \rightarrow a^1\Pi_g$	<b>404.8</b>	<b>345.6</b>	<b>343.2</b>
	$\rightarrow a'^1\Sigma_u^-$	<b>375.9</b>	<b>367.9</b>	<b>367.0</b>
	$\rightarrow w^1\Delta_u$	412.1	383.5	381.7
<b>1.2700</b>	$X^1\Sigma_g^+ \rightarrow a^1\Pi_g$	313.8	274.6	272.2
	$\rightarrow a'^1\Sigma_u^-$	280.6	271.6	270.6
	$\rightarrow w^1\Delta_u$	336.9	290.3	288.2
<b>1.4288</b>	$X^1\Sigma_g^+ \rightarrow a^1\Pi_g$	252.8	226.7	223.4
	$\rightarrow a'^1\Sigma_u^-$	215.9	207.0	206.2
<b>RMS Error</b>		<b>35.3</b>	<b>2.0</b>	

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## VIII. CBS Extrapolations

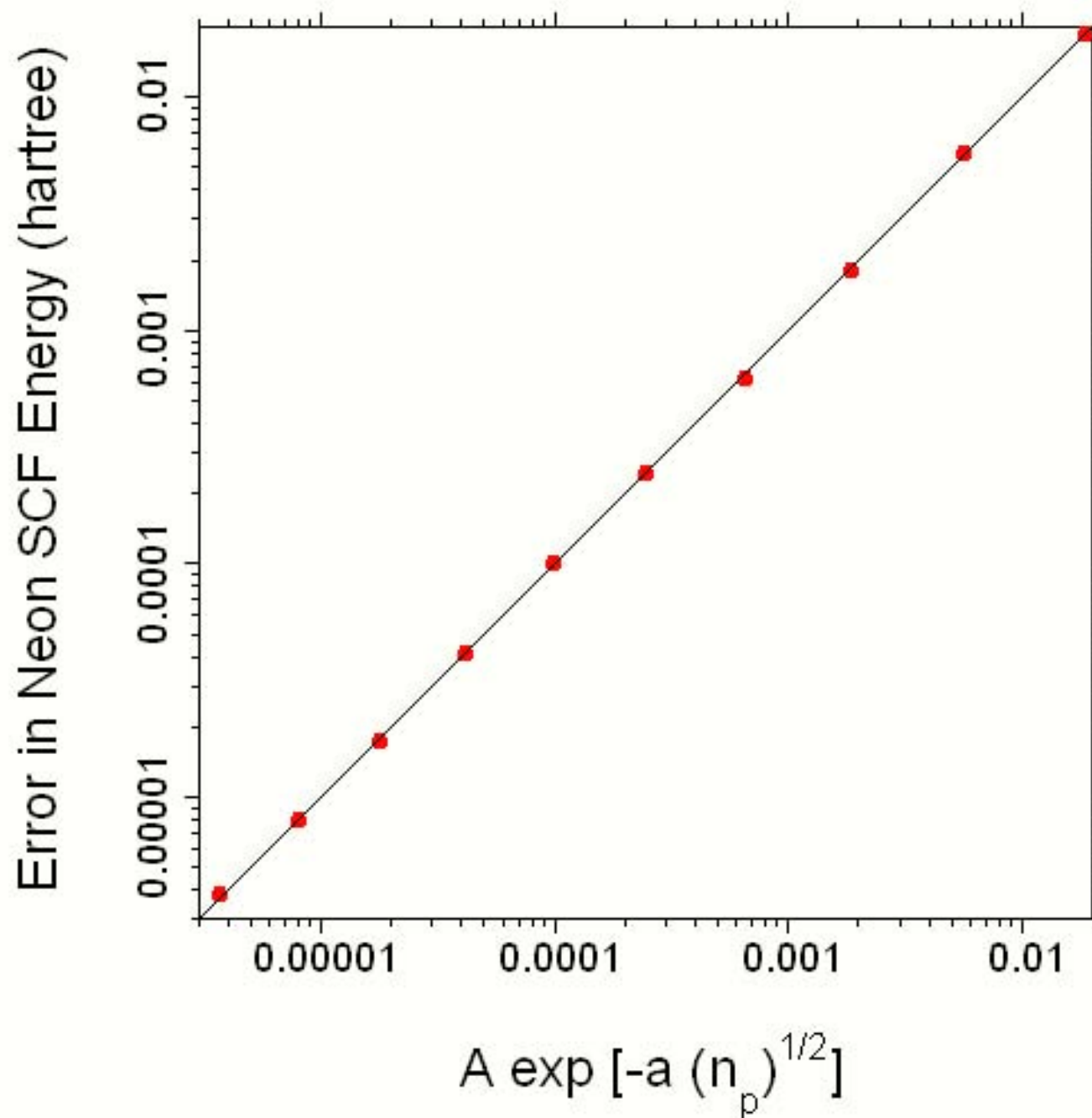
- a. A Systematic Sequence of Basis Sets
- b. The CASSCF CBS Limit
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## X. Acknowledgment

The **SCF** energy converges to the **CBS** (complete basis set) limit as the exponential of the square-root of the number of Gaussian basis functions.



$$E_{SCF \text{ Limit}} \cong E(n_2) + \{ \exp[a(n_1)^{1/2} - a(n_2)^{1/2}] - 1 \}^{-1} \{ E(n_2) - E(n_1) \}$$

**Table III.** Linear SCF extrapolation parameters for the neon atom.

$n_1$	$n_2$	$\{ \exp[a(n_1)^{1/2} - a(n_2)^{1/2}] - 1 \}^{-1}$
5	6	0.426
6	7	0.490
7	8	0.551
8	9	0.609
<b>6</b>	<b>8</b>	<b>0.132</b>
<b>8</b>	<b>10</b>	<b>0.178</b>
<b>10</b>	<b>12</b>	<b>0.220</b>
<b>12</b>	<b>14</b>	<b>0.262</b>

**Table IV.** The convergence of the N<sub>2</sub> SCF anisotropy energy (hartree) with the number of polarization functions in the basis set.

Basis Set	E <sub>SCF</sub>	ΔE <sub>SCF</sub> (n)	E <sub>SCF</sub> Error	Extrapolation Factor <sup>a</sup>
16s10p	-108.910331		0.082734	
16s10p1d	-108.981649	-0.071318	-0.011416	
16s10p2d1f	-108.991667	-0.010018	-0.001398	<b>0.139</b>
16s10p3d2f1g	-108.992847	-0.001180	-0.000218	<b>0.184</b>
16s10p4d3f2g1h	-108.993025	-0.000178	-0.000040	<b>0.22</b>
16s10p5d4f3g2h1i	-108.993057	-0.000032	-0.000008	<b>0.25</b>
SCF Limit = -108.993065				

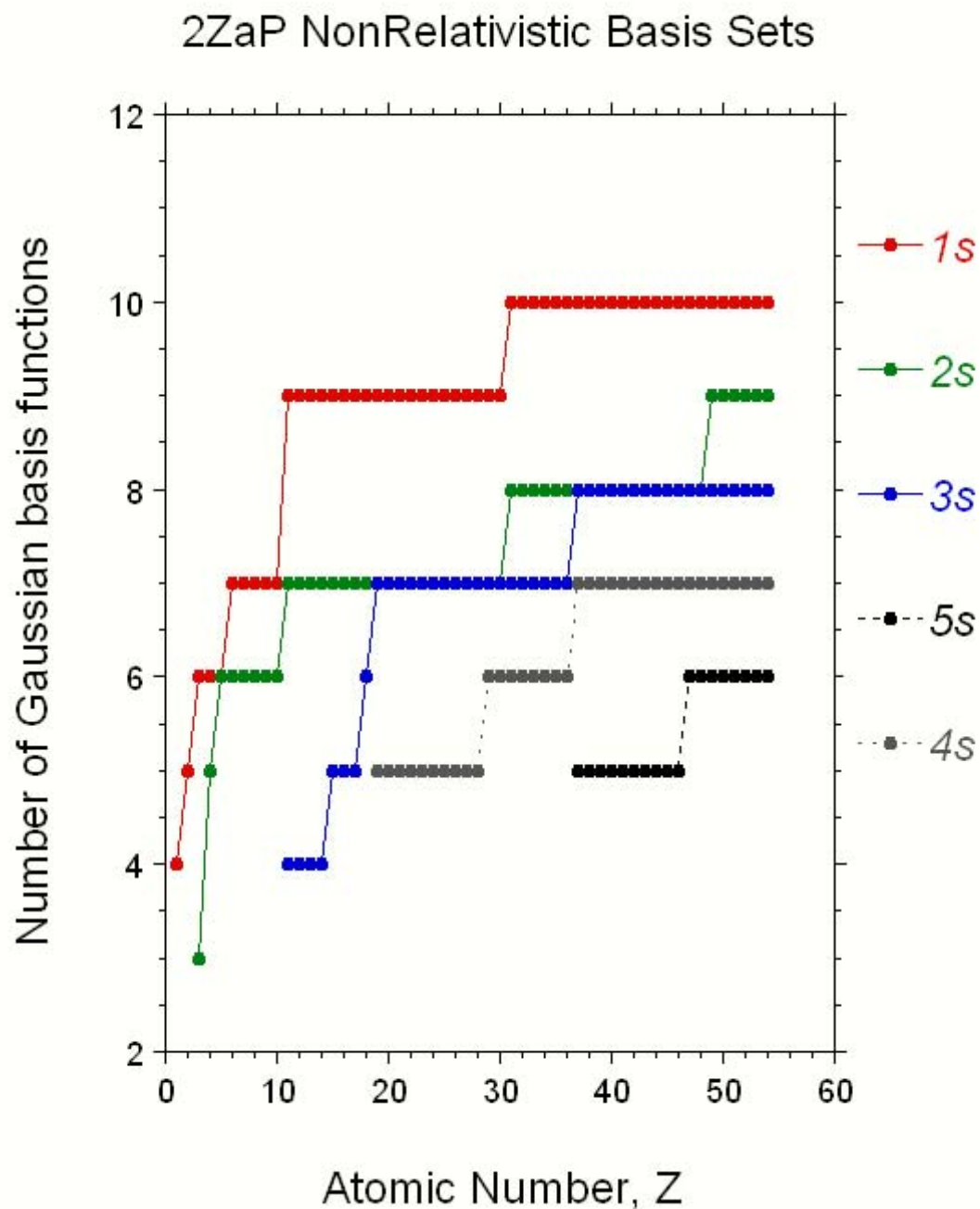
a. Factor = E<sub>SCF</sub> Error(n) / ΔE<sub>SCF</sub>(n)

## The nZaP BasisSets

The number of Gaussian primitives in the 2ZaP basis sets was selected to give a constant, *1 millihartree, SCF error per electron*.

The 3ZaP basis sets each contain two more primitives of each angular momentum.

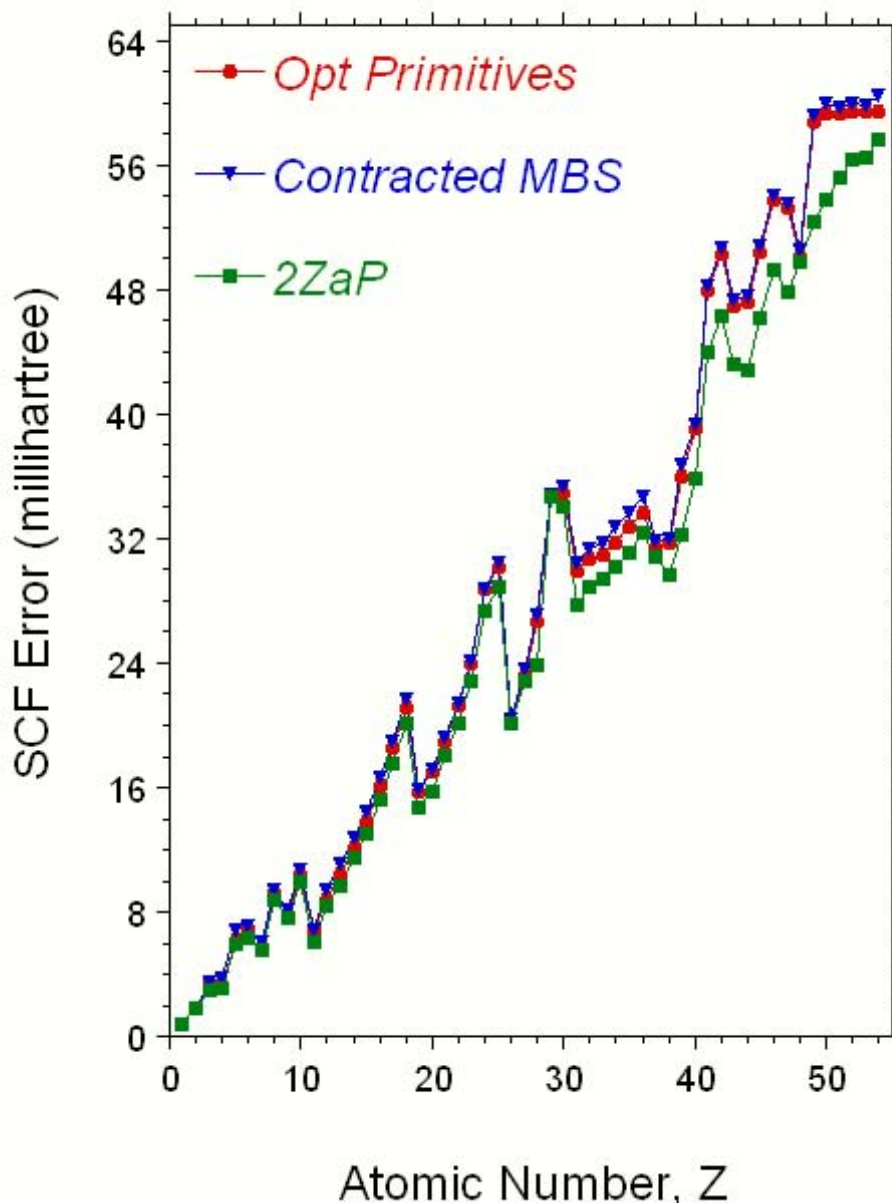
The 4ZaP basis sets include an additional two, etc.



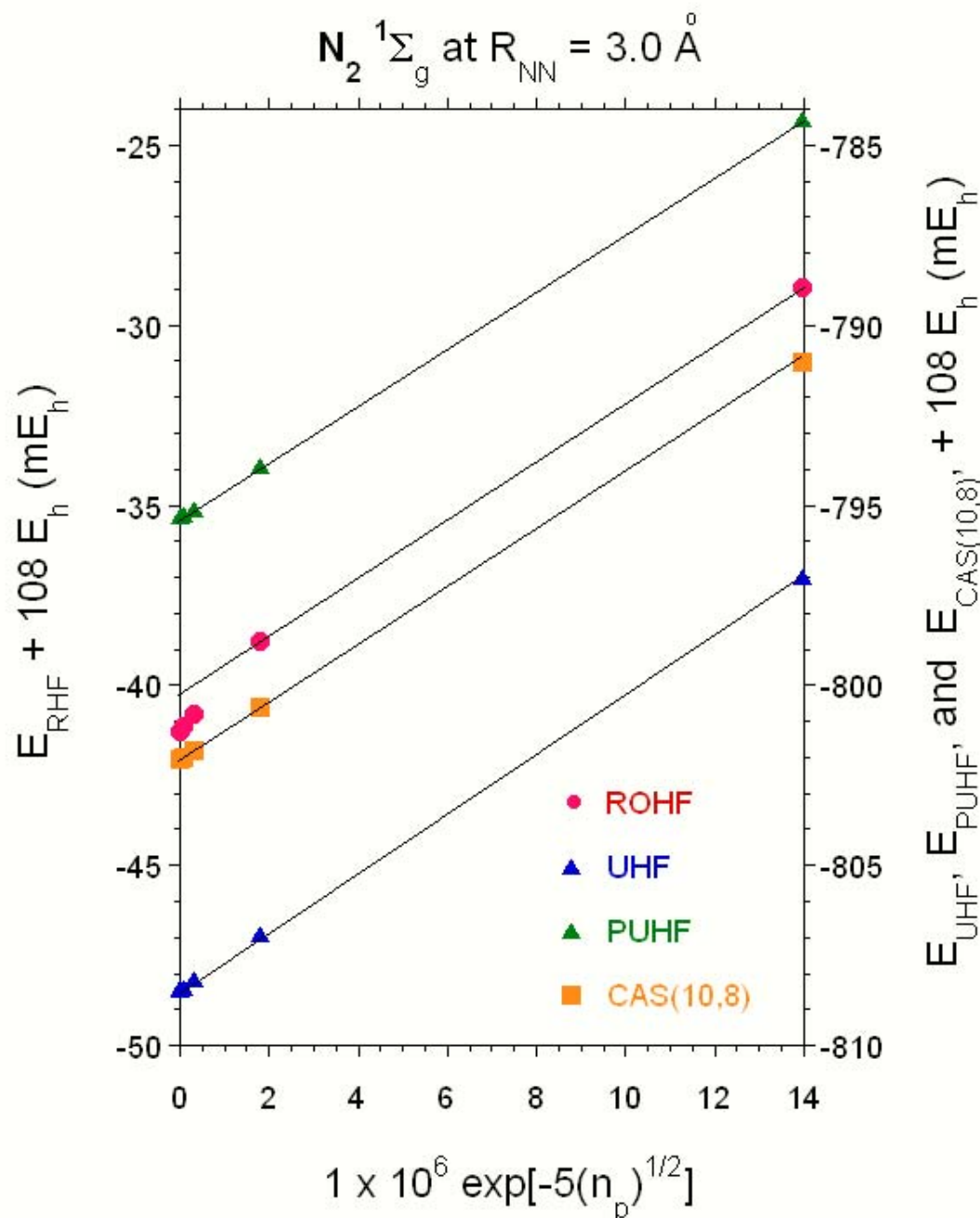


## The nZaP BasisSets

The contracted nZaP basis sets are similar in structure to the cc-pVnZ basis sets, but with *diffuse valence functions added*. The number of primitives in each contracted function is sufficient to prevent contraction from significantly raising the SCF energy. All “correlation primitives” (*valence and polarization*) were optimized at the MP2 level.



**UHF** energies  
mimic the  
basis set  
convergence  
of **CASSCF**  
energies, but  
**ROHF**  
energies  
do not.



**Table V.** Basis set convergence of CASSCF and UHFSCF as a function of geometry.

$R_{NN}$ (Å)	Basis Set	CAS(6,6) ( $E_h$ )	$\Delta E$ ( $mE_h$ )	$\Delta(\Delta E)_R$ ( $mE_h$ )	UHF ( $E_h$ )	$\Delta E$ ( $mE_h$ )	$\Delta(\Delta E)_R$ ( $mE_h$ )
1.09	2ZaP	-109.1035124			-108.9687667		
	3ZaP	-109.1246270	-21.1146	0.5215	-108.9903071	-21.5404	0.4654
	4ZaP	-109.1279935	-3.3665	0.1042	-108.9935983	-3.2912	0.0966
	5ZaP	-109.1288265	-0.8330	0.0213	-108.9944093	-0.8110	0.0198
	6ZaP				-108.9945297	-0.1204	0.0013
1.10	2ZaP	-109.1045238			-108.9674580		
	3ZaP	-109.1251169	<b>-20.5931</b>	<b>0.4819</b>	-108.9885330	<b>-21.0750</b>	<b>0.4264</b>
	4ZaP	-109.1283792	<b>-3.2623</b>	<b>0.0978</b>	-108.9917276	<b>-3.1946</b>	<b>0.0898</b>
	5ZaP	-109.1291909	<b>-0.8117</b>	<b>0.0204</b>	-108.9925188	<b>-0.7912</b>	<b>0.0189</b>
	6ZaP				-108.9926379	-0.1191	0.0011
1.11	2ZaP	-109.1049575			-108.9655455		
	3ZaP	-109.1250687	-20.1112	0.4447	-108.9861941	-20.6486	0.3898
	4ZaP	-109.1282332	-3.1645	0.0917	-108.9892989	-3.1048	0.0833
	5ZaP	-109.1290245	-0.7913	0.0195	-108.9900712	-0.7723	0.0181
	6ZaP				-108.9901892	-0.1180	0.0011

$$\begin{aligned}
E_{CAS}(CBS) &\cong E_{CAS}(3ZaP) \\
&+ 1.2 \times [E_{UHF}(4ZaP) - E_{UHF}(3ZaP)] \\
&\times \left[ \frac{E_{CAS}(3ZaP) - E_{CAS}(2ZaP)}{E_{UHF}(3ZaP) - E_{UHF}(2ZaP)} \right]
\end{aligned}$$

**Table VI.** Calculated Full Valence CASSCF energies (hartree au).

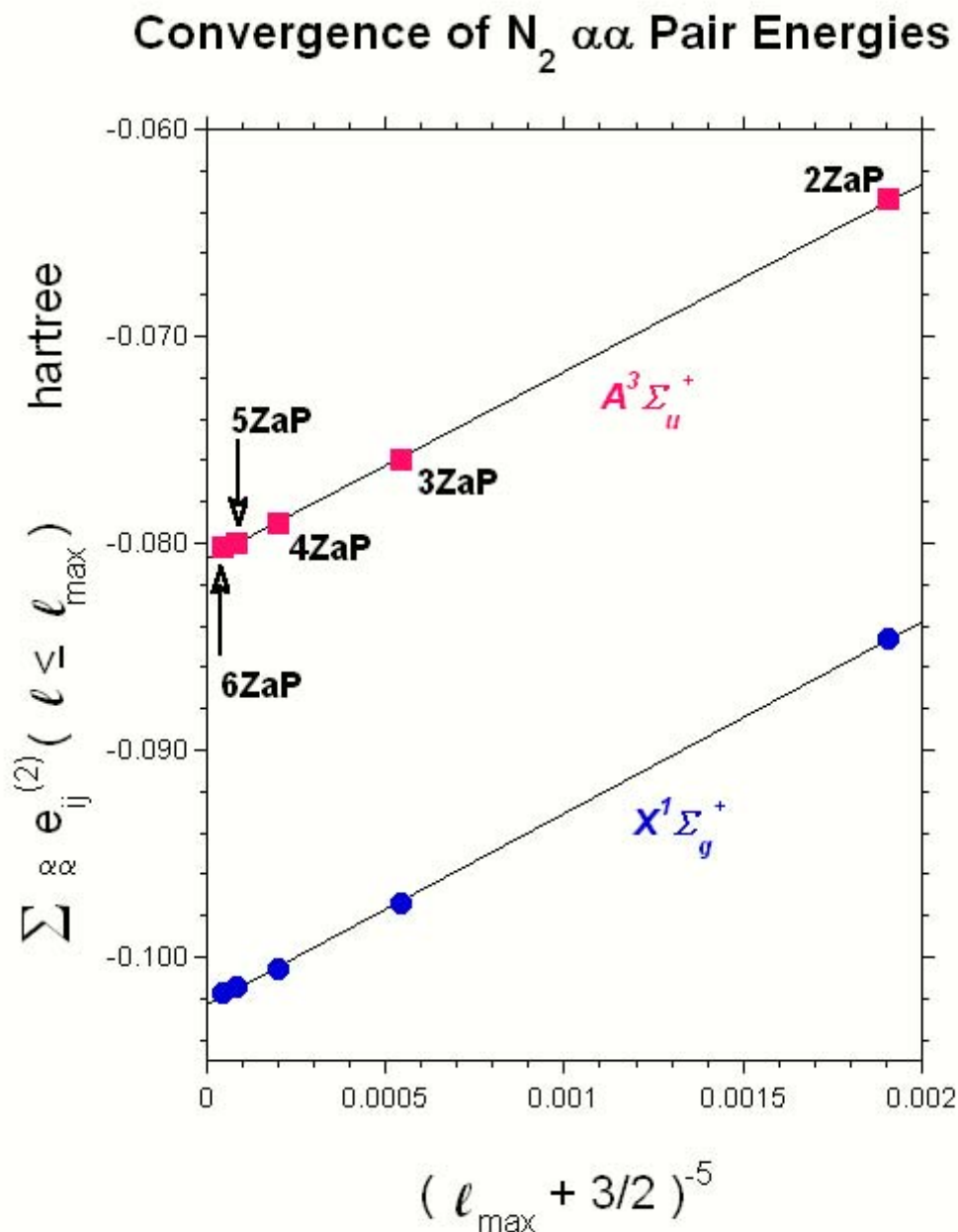
	<b>2ZaP</b>	<b>3ZaP</b>	<b>4ZaP</b>	<b>5ZaP</b>	<b>6ZaP</b>	<b>Limit</b>
<b>C<sub>2</sub> X<sup>1</sup>Σ<sub>g</sub><sup>+</sup></b>	-75.6233832	-75.6412898	-75.6437447	-75.6442121	-75.6442908	-75.6443188
<b>C<sub>2</sub> a<sup>3</sup>Π<sub>u</sub></b>	-75.6073350	-75.6230710	-75.6251470	-75.6255495	-75.6256266	-75.6256541
<b>C<sub>2</sub> b<sup>3</sup>Σ<sub>g</sub><sup>-</sup></b>	-75.5777931	-75.5941905	-75.5962600	-75.5966964	-75.5967840	-75.5968152
<b>C<sub>2</sub> A<sup>1</sup>Π<sub>u</sub></b>	-75.5627890	-75.5788475	-75.5809697	-75.5813798	-75.5814615	-75.5814906
<b>CN X<sup>2</sup>Σ<sup>+</sup></b>	-92.3544025	-92.3730997	-92.3761806	-92.3766716	-92.3767524	-92.3767812
<b>CO X<sup>1</sup>Σ<sup>+</sup></b>	-112.8946223	-112.9181416	-112.9218328	-112.9224855	-112.9226137	-112.9226594
<b>N<sub>2</sub> X<sup>1</sup>Σ<sub>g</sub><sup>+</sup></b>	-109.1166132	-109.1373292	-109.1406653	-109.1414826	-109.1415931	-109.1416324
<b>NO X<sup>2</sup>Π<sub>i</sub></b>	-129.3892421	-129.4106758	-129.4138042	-129.4143983	-129.4144941	-129.4145282
<b>O<sub>2</sub> X<sup>3</sup>Σ<sub>g</sub><sup>-</sup></b>	-149.7411441	-149.7681917	-149.7713131	-149.7719475	-149.7720640	-149.7721055
<b>O<sub>2</sub> a<sup>1</sup>Δ<sub>g</sub></b>	-149.7059304	-149.7327122	-149.7358092	-149.7364403	-149.7365522	-149.7365920
<b>O<sub>3</sub> X<sup>1</sup>A<sub>1</sub></b>	-224.5478659	-224.5938866	-224.5991383	-224.6002546	-224.6004950	-224.6005806
<b>OH X<sup>2</sup>Π</b>	-75.4325178	-75.4440618	-75.4456305	-75.4458720	-75.4459251	-75.4459441
<b>OH A<sup>2</sup>Σ<sup>+</sup></b>	-75.2670507	-75.2808979	-75.2825953	-75.2828559	-75.2829216	-75.2829450
<b>RMS Error</b>	<b>0.0224147</b>	<b>0.0030328</b>	<b>0.0005829</b>	<b>0.0001180</b>	<b>0.0000310</b>	

**Table VII.** Extrapolated Full Valence CASSCF energies (hartree au).

	<b>3ZaP</b>	<b>Error 4ZaP</b>	<b>5ZaP</b>
<b>C<sub>2</sub> X<sup>1</sup>Σ<sub>g</sub><sup>+</sup>(Re)</b>	-0.0001705	0.0000255	-0.0000059
<b>C<sub>2</sub> a<sup>3</sup>Π<sub>u</sub>(Re)</b>	-0.0001753	0.0000582	0.0000012
<b>C<sub>2</sub> b<sup>3</sup>Σ<sub>g</sub><sup>-</sup>(Re)</b>	-0.0002094	0.0000064	-0.0000040
<b>C<sub>2</sub> A<sup>1</sup>Π<sub>u</sub>(Re)</b>	-0.0000459	0.0000176	0.0000026
<b>CN X<sup>2</sup>Σ<sup>+</sup>(Re)</b>	-0.0001536	0.0000366	-0.0000031
<b>N<sub>2</sub> X<sup>1</sup>Σ<sub>g</sub><sup>+</sup>(Re)</b>	-0.0005054	-0.0001428	-0.0000276
<b>NO X<sup>2</sup>Π<sub>i</sub>(Re)</b>	-0.0001124	-0.0001120	-0.0000175
<b>O<sub>2</sub> X<sup>3</sup>Σ<sub>g</sub><sup>-</sup>(Re)</b>	0.0000564	-0.0001285	-0.0000186
<b>O<sub>2</sub> a<sup>1</sup>Δ<sub>g</sub>(Re)</b>	-0.0000230	-0.0001280	-0.0000168
<b>OH X<sup>2</sup>Π(Re)</b>	-0.0000524	-0.0000353	-0.0000136
<b>OH A<sup>2</sup>Σ<sup>+</sup>(Re)</b>	0.0000168	-0.0000603	-0.0000194
<b>RMS Error</b>	<b>0.0002327</b>	<b>0.0000956</b>	<b>0.0000245</b>
<b>Before Extrap</b>	<b>0.0030328</b>	<b>0.0005829</b>	<b>0.0001180</b>

## MP2 CBS Limit

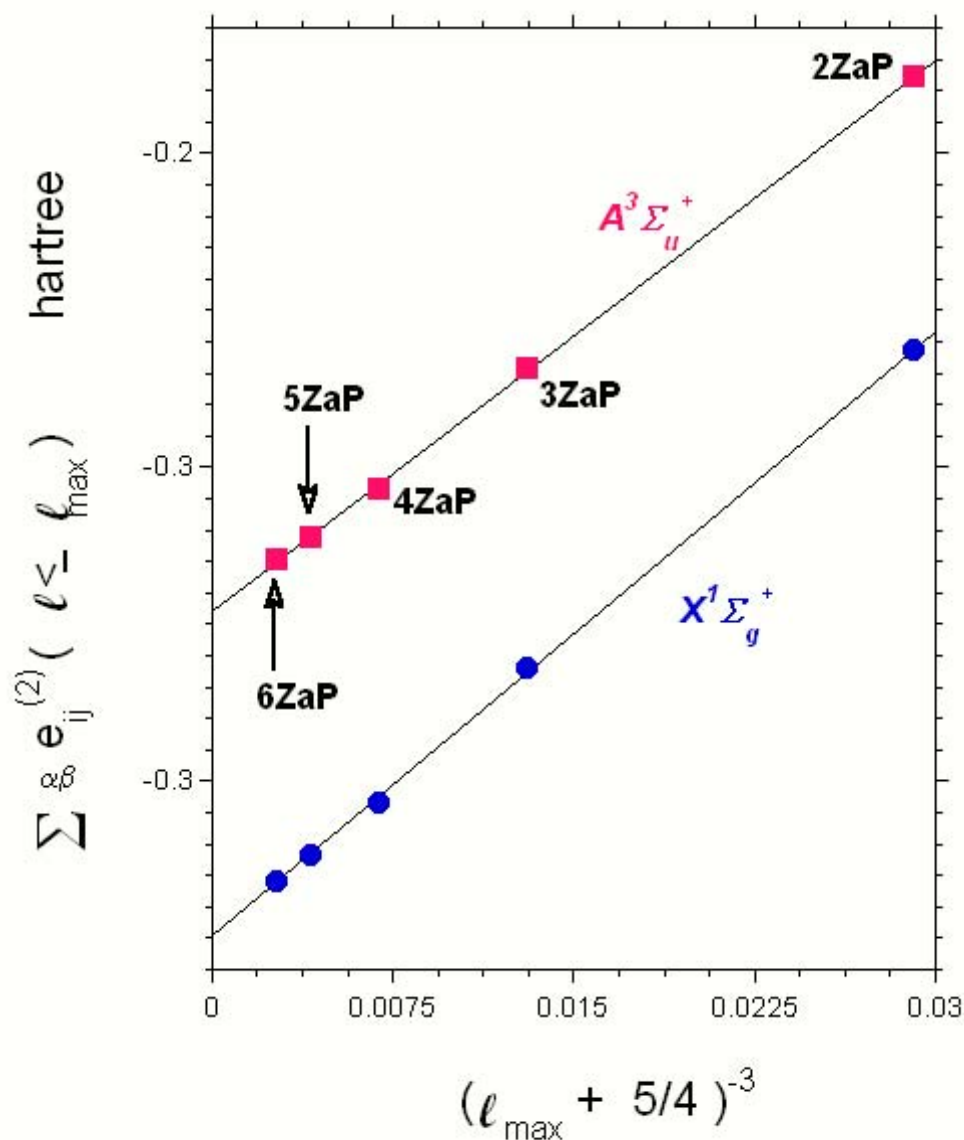
The  $\alpha\alpha$  second-order Møller-Plessett electron pair correlation energies converge to the CBS limit as the inverse **fifth** power of the maximum angular momentum included in the basis set.



## MP2 CBS Limit

The  $\alpha\beta$  second-order Møller-Plessett electron pair correlation energies converge to the CBS limit as the inverse **third** power of the maximum angular momentum included in the basis set.

Convergence of  $N_2$   $\alpha\beta$  Pair Energies





**Table VIII.** The basis set convergence of dynamic correlaton energy.

State	Basis Set	$E^{(2)}$ ( $mE_h$ )	$\Delta E^{(2)}$ ( $mE_h$ )	$E_{\text{CAS-CISD}} - E_{\text{CAS}}$ ( $mE_h$ )	$\Delta(E_{\text{CAS-CISD}} - E_{\text{CAS}})$ ( $mE_h$ )
$X^1\Sigma_g^+$ 1.10Å	2ZaP	-316.0189		-181.9089	
	3ZaP	-379.3504	<b>-63.3315</b>	-244.4998	<b>-62.5909</b>
	4ZaP	-403.7559			
	5ZaP	-413.3317			
	6ZaP	-417.4648			
	CBS Limit	-426.90			
$A^3\Sigma_u^+$ 1.27Å	2ZaP	-250.9989		-195.6356	
	3ZaP	-310.1058	<b>-59.1069</b>	-255.0882	<b>-59.4526</b>
	4ZaP	-332.4087			
	5ZaP	-341.1121			
	6ZaP	-344.7652			
	CBS Limit	-353.59			

$$\begin{aligned}
E_{CAS-CISD}(CBS) &\cong E_{CAS-CISD}(3ZaP) \\
&+ \left[ E^{(2)}(CBS) - E^{(2)}(3ZaP) \right] \\
&\times \left[ \frac{E_{CAS-CISD}(3ZaP) - E_{CAS-CISD}(2ZaP)}{E^{(2)}(3ZaP) - E^{(2)}(2ZaP)} \right]
\end{aligned}$$

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## **XIII. CBS Extrapolations**

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## **XIV. Results**

- a. Geometry
- b. Vibrational Frequencies
- c. Excitation energies
- d. Dissociation Energies

## **XV. Acknowledgment**

**"Approved for public release; distribution unlimited"**

**Table IX.** Calculated CAS-CISD **geometries** ( $R_e$  in Å) for low-lying excited states of  $N_2$ .

<b>State</b>	<b>2ZaP</b>	<b>3ZaP</b>	<b>CBS limit</b>	<b>Exp.</b>
$X^1\Sigma_g^+$	1.1175	1.1039	1.0991	1.0977
$A^3\Sigma_u^+$	1.3066	1.2951	1.2905	1.2866
$B^3\Pi_g$	1.2314	1.2201	1.2152	1.2126
$W^3\Delta_u$	1.2972	1.2859	1.2815	
$B'^3\Sigma_u^-$	1.2955	1.2845	1.2800	1.2784
$a'^1\Sigma_u^-$	1.2914	1.2805	1.2759	1.2755
$a^1\Pi_g$	1.2390	1.2271	1.2213	1.2203
$w^1\Delta_u$	1.2837	1.2728	1.2684	1.268
<b>RMS Error</b>	<b>0.0181</b>	<b>0.0066</b>	<b>0.0020</b>	

**Table X.** Calculated CAS-CISD harmonic **vibrational frequencies** ( $\omega_e$  in  $\text{cm}^{-1}$ ) for low-lying excited states of  $\text{N}_2$ .

<b>State</b>	<b>2ZaP</b>	<b>3ZaP</b>	<b>CBS limit</b>	<b>Exp.</b>
$X^1\Sigma_g^+$	2319.1	2345.2	2358.1	2358.57
$A^3\Sigma_u^+$	1418.5	1439.7	1450.8	1460.64
$B^3\Pi_g$	1707.4	1715.3	1727.4	1733.39
$W^3\Delta_u$	1482.0	1499.1	1508.9	1501.4
$B'^3\Sigma_u^-$	1494.5	1509.9	1518.9	1516.88
$a'^1\Sigma_u^-$	1508.6	1528.4	1538.8	1530.25
$a^1\Pi_g$	1648.2	1686.1	1696.4	1694.21
$w^1\Delta_u$	1543.5	1561.3	1571.3	1559.26
<b>RMS Error</b>	<b>31.1</b>	<b>12.1</b>	<b>7.3</b>	

**Table XI.** Calculated CAS-CISD **excitation energies** (Ev) of N<sub>2</sub>.

<b>State</b>	<b>2ZaP</b>	<b>3ZaP</b>	<b>CBS limit</b>	<b>Exp.</b>
$X^1\Sigma_g^+ \rightarrow$				
$A^3\Sigma_u^+$	5.843	6.093	6.192	6.224
$B^3\Pi_g$	7.275	7.397	7.447	7.392
$W^3\Delta_u$	7.233	7.364	7.432	7.415
$B'^3\Sigma_u^-$	8.067	8.230	8.265	8.217
$a'^1\Sigma_u^-$	8.427	8.471	8.407	8.450
$a^1\Pi_g$	8.547	8.653	8.562	8.590
$w^1\Delta_u$	8.932	9.008	8.927	8.939
<b>RMS Error</b>	<b>0.176</b>	<b>0.065</b>	<b>0.037</b>	

**Table XII.** Calculated CAS-CISD dissociation energies (Ev) of N<sub>2</sub>.

<b>State</b>	<b>2ZaP</b>	<b>3ZaP</b>	<b>CBS limit</b>	<b>Exp.</b>
$X^1\Sigma_g^+ \rightarrow$				
$A^3\Sigma_u^+$	5.857	6.093	6.192	6.224
$B^3\Pi_g$	7.256	7.397	7.447	7.392
$W^3\Delta_u$	7.301	7.364	7.432	7.415
$B'^3\Sigma_u^-$	8.063	8.230	8.265	8.217
$a'^1\Sigma_u^-$	8.417	8.471	8.407	8.450
$a^1\Pi_g$	8.537	8.653	8.562	8.590
$w^1\Delta_u$	8.914	9.008	8.927	8.939
<b>RMS Error</b>	<b>0.167</b>	<b>0.065</b>	<b>0.037</b>	

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